Chapter 2

Measurement and Problem Solving

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The graph in this image displays average global temperatures (relative to the mean) over the past 100 years.
The uncertainty is indicated by the last reported digit.

Example: measuring global temperatures

Average global temperatures have risen by 0.6 °C in the last century.

By reporting a temperature increase of 0.6 °C, the scientists mean 0.6 +/− 0.1 °C.

The temperature rise could be as much as 0.7 °C or as little as 0.5 °C, but it is not 1.0 °C.

The degree of certainty in this particular measurement is critical, influencing political decisions that directly affect people’s lives.
A number written in scientific notation has two parts.

A decimal part: a number that is between 1 and 10.

An exponential part: 10 raised to an exponent, $n$. 

\[ 1.2 \times 10^{-10} \]
Writing Very Large and Very Small Numbers

• A positive exponent means 1 multiplied by $10^n$ times.
• A negative exponent ($-n$) means 1 divided by $10^n$ times.

$$10^0 = 1$$
$$10^1 = 1 \times 10 = 10$$
$$10^2 = 1 \times 10 \times 10 = 100$$
$$10^3 = 1 \times 10 \times 10 \times 10 = 1000$$

$$10^{-1} = \frac{1}{10} = 0.1$$
$$10^{-2} = \frac{1}{10 \times 10} = 0.01$$
$$10^{-3} = \frac{1}{10 \times 10 \times 10} = 0.001$$
To Convert a Number to Scientific Notation

• Find the decimal part. Find the exponent.
• Move the decimal point to obtain a number between 1 and 10.
• Multiply that number (the decimal part) by 10 raised to the power that reflects the movement of the decimal point.
To Convert a Number to Scientific Notation

5983 = 5.983 \times 10^3 \\
0.00034 = 3.4 \times 10^{-4}

• If the decimal point is moved to the left, the exponent is positive.
• If the decimal point is moved to the right, the exponent is negative.
Pennies come in whole numbers, and a count of seven pennies means seven whole pennies.

Our knowledge of the amount of gold in a 10-g gold bar depends on how precisely it was measured.
Reporting Scientific Numbers

The first four digits are certain; the last digit is estimated.

The greater the precision of the measurement, the greater the number of significant figures.
Estimating Tenths of a Gram

- This balance has markings every 1 g.
- We estimate to the tenths place.
- To estimate between markings, mentally divide the space into 10 equal spaces and estimate the last digit.
- This reading is 1.2 g.
Estimating Hundredths of a Gram

- This scale has markings every 0.1 g.
- We estimate to the hundredths place.
- The correct reading is 1.26 g.
Significant Figures in a Correctly Reported Measurement

1. All **nonzero** digits **are significant**.
2. **Interior zeros** (zeros between two numbers) **are significant**.
3. **Trailing zeros** (zeros to the right of a nonzero number) that fall after a decimal point **are significant**.
4. **Trailing zeros** that fall before a decimal point **are significant**.
5. **Leading zeros** (zeros to the left of the first nonzero number) **are NOT significant**. They serve only to locate the decimal point.
6. **Trailing zeros at the end** of a number, but before an implied decimal point, **are ambiguous** and should be avoided.
Identifying Exact Numbers

- Exact numbers have an unlimited number of significant figures.
- Exact counting of discrete objects
- Integral numbers that are part of an equation
- Defined quantities
- Some conversion factors are defined quantities, while others are not.

1 in. = 2.54 cm exact
Counting Significant Figures

How many significant figures are in each number?

0.0035  two significant figures
1.080    four significant figures
2371     four significant figures
2.9 \times 10^5 three significant figures
1 \text{ dozen} = 12 unlimited significant figures
100.00   five significant figures
100,000  ambiguous
Rules for Rounding:

- When numbers are used in a calculation, the result is rounded to reflect the significant figures of the data.

- For calculations involving multiple steps, round only the final answer—do not round off between steps. This practice prevents small rounding errors from affecting the final answer.
Rules for Rounding:

- Use only the last (or leftmost) digit being dropped to decide in which direction to round—ignore all digits to the right of it.

- Round down if the last digit dropped is 4 or less; round up if the last digit dropped is 5 or more.
Multiplication and Division Rule:

The result of multiplication or division carries the same number of significant figures as the factor with the fewest significant figures.
Multiplication and Division Rule:

The intermediate result (in blue) is rounded to two significant figures to reflect the least precisely known factor (0.10), which has two significant figures.
Multiplication and Division Rule:

The intermediate result (in blue) is rounded to three significant figures to reflect the least precisely known factor (6.10), which has three significant figures.
Addition and Subtraction Rule:

In addition or subtraction calculations, the result carries the same number of decimal places as the quantity carrying the fewest decimal places.
**Significant Figures in Calculations**

**Addition and Subtraction Rule:**

We round the intermediate answer (in blue) to two decimal places because the quantity with the fewest decimal places (5.74) has two decimal places.
Significant Figures in Calculations

Addition and Subtraction Rule:

We round the intermediate answer (in blue) to one decimal place because the quantity with the fewest decimal places (4.8) has one decimal place.
In calculations involving both multiplication/division and addition/subtraction, do the steps in parentheses first; determine the correct number of significant figures in the intermediate answer without rounding; then do the remaining steps.
In the calculation $3.489 \times (5.67 - 2.3)$, do the step in parentheses first. $5.67 - 2.3 = 3.37$
Use the subtraction rule to determine that the intermediate answer has only one significant decimal place.
To avoid small errors, it is best not to round at this point; instead, underline the least significant figure as a reminder.

$3.489 \times 3.37 = 11.758 = 12$

Use the multiplication rule to determine that the intermediate answer (11.758) rounds to two significant figures (12) because it is limited by the two significant figures in 3.37.
The unit system for science measurements, based on the metric system, is called the **International System of Units (Système International d’Unités)** or **SI units**.

### TABLE 2.1 Important SI Base Units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>meter</td>
<td>m</td>
</tr>
<tr>
<td>mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>temperature*</td>
<td>kelvin</td>
<td>K</td>
</tr>
</tbody>
</table>

*Temperature units are discussed in Chapter 3.
The standard of length The definition of a meter, established by international agreement in 1983, is the distance that light travels in vacuum in $1/299,792,458$ s. (The speed of light is $299,792,458$ m/s.)
• **The standard of mass**
  The kilogram is defined as the mass of a block of metal kept at the International Bureau of Weights and Measures at Sèvres, France. A duplicate is kept at the National Institute of Standards and Technology near Washington, D.C.
• **The standard of time** The second is defined, using an atomic clock, as the duration of 9,192,631,770 periods of the radiation emitted from a certain transition in a cesium-133 atom.
Weight vs. Mass

• The kilogram is a measure of mass, which is different from weight.
• The mass of an object is a measure of the quantity of matter within it.
• The weight of an object is a measure of the gravitational pull on that matter.
• Consequently, weight depends on gravity while mass does not.
# SI Prefix Multipliers

## TABLE 2.2 SI Prefix Multipliers

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Meaning</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>tera-</td>
<td>T</td>
<td>trillion</td>
<td>1,000,000,000,000,000</td>
</tr>
<tr>
<td>giga-</td>
<td>G</td>
<td>billion</td>
<td>1,000,000,000</td>
</tr>
<tr>
<td>mega-</td>
<td>M</td>
<td>million</td>
<td>1,000,000</td>
</tr>
<tr>
<td>kilo-</td>
<td>k</td>
<td>thousand</td>
<td>1,000</td>
</tr>
<tr>
<td>hecto-</td>
<td>h</td>
<td>hundred</td>
<td>100</td>
</tr>
<tr>
<td>deca-</td>
<td>da</td>
<td>ten</td>
<td>10</td>
</tr>
<tr>
<td>deci-</td>
<td>d</td>
<td>tenth</td>
<td>0.1</td>
</tr>
<tr>
<td>centi-</td>
<td>c</td>
<td>hundredth</td>
<td>0.01</td>
</tr>
<tr>
<td>milli-</td>
<td>m</td>
<td>thousandth</td>
<td>0.001</td>
</tr>
<tr>
<td>micro-</td>
<td>μ</td>
<td>millionth</td>
<td>0.000001</td>
</tr>
<tr>
<td>nano-</td>
<td>n</td>
<td>billionth</td>
<td>0.00000000001</td>
</tr>
<tr>
<td>pico-</td>
<td>p</td>
<td>trillionth</td>
<td>0.000000000000001</td>
</tr>
<tr>
<td>femto-</td>
<td>f</td>
<td>quadrillionth</td>
<td>0.00000000000000001</td>
</tr>
</tbody>
</table>
• Choose the prefix multiplier that is most convenient for a particular measurement.
• Pick a unit similar in size to (or smaller than) the quantity you are measuring.
• A short chemical bond is about $1.2 \times 10^{-10}$ m. Which prefix multiplier should you use?
  
  \begin{align*}
    \text{pico} &= 10^{-12}; \\
    \text{nano} &= 10^{-9}
  \end{align*}

• The most convenient one is probably the picometer. Chemical bonds measure about 120 pm.
Volume as a Derived Unit

• A derived unit is formed from other units.
• Many units of **volume**, a measure of space, are derived units.
• Any unit of length, when cubed (raised to the third power), becomes a unit of volume.
• Cubic meters (m$^3$), cubic centimeters (cm$^3$), and cubic millimeters (mm$^3$) are all units of volume.
Getting to an equation to solve from a problem statement requires critical thinking. No simple formula applies to every problem, yet you can learn problem-solving strategies and begin to develop some chemical intuition.

Unit conversion type:

Many of the problems can be thought of as unit conversion problems, in which you are given one or more quantities and asked to convert them into different units.

Specific equation type:

Other problems require the use of specific equations to get to the information you are trying to find.
Units are multiplied, divided, and canceled like any other algebraic quantities.

Using units as a guide to solving problems is called dimensional analysis.

Always write every number with its associated unit.

Always include units in your calculations, dividing them and multiplying them as if they were algebraic quantities.

Do not let units appear or disappear in calculations. Units must flow logically from beginning to end.
• For most conversion problems, we are given a quantity in some units and asked to convert the quantity to another unit. These calculations take the form:

\[
\text{information given} \times \text{conversion factor(s)} = \text{information sought}
\]

\[
\frac{\text{given unit}}{\text{given unit}} \times \frac{\text{desired unit}}{\text{given unit}} = \text{desired unit}
\]
Converting Between Units

- Conversion factors are constructed from any two quantities known to be equivalent.
- We construct the conversion factor by dividing both sides of the equality by 1 in. and canceling the units.

\[
\frac{2.54 \text{ cm}}{1 \text{ in.}} = \frac{1 \text{ in.}}{1 \text{ in.}}
\]

\[
\frac{2.54 \text{ cm}}{1 \text{ in.}} = 1
\]

The quantity \(\frac{2.54 \text{ cm}}{1 \text{ in.}}\) equal to 1 and can be used to convert between inches and centimeters.
Converting Between Units

• In solving problems, always check if the final units are correct, and consider whether or not the magnitude of the answer makes sense.

\[
44.7 \text{ cm} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} = 17.6 \text{ in.}
\]

• Conversion factors can be inverted because they are equal to 1 and the inverse of 1 is 1.

\[
\frac{1}{1} = 1
\]

Therefore,

\[
\frac{2.54 \text{ cm}}{1 \text{ in.}} = 1 = \frac{1 \text{ in.}}{2.54 \text{ cm}}
\]
The Solution Map

• A solution map is a visual outline that shows the strategic route required to solve a problem.

• For unit conversion, the solution map focuses on units and how to convert from one unit to another.
Diagram Conversions Using a Solution Map

- The solution map for converting from inches to centimeters is as follows:

  ![Diagram for inches to centimeters conversion]

  \[
  \frac{2.54 \text{ cm}}{1 \text{ in.}}
  \]

- The solution map for converting from centimeters to inches is as follows:

  ![Diagram for centimeters to inches conversion]

  \[
  \frac{1 \text{ in.}}{2.54 \text{ cm}}
  \]
General Problem-Solving Strategy

- Identify the starting point (the *given* information).
- Identify the end point (what you must *find*).
- Devise a way to get from the starting point to the end point using what is given as well as what you already know or can look up.
- You can use a *solution map* to diagram the steps required to get from the starting point to the end point.
- In graphic form, we can represent this progression as

  **Given** → **Solution Map** → **Find**
General Problem-Solving Strategy

- Sort. Begin by sorting the information in the problem.
- Strategize. Create a solution map—the series of steps that will get you from the given information to the information you are trying to find.
- Solve. Carry out mathematical operations (paying attention to the rules for significant figures in calculations) and cancel units as needed.
- Check. Does this answer make physical sense? Are the units correct?
• Each step in the solution map should have a conversion factor with the units of the previous step in the denominator and the units of the following step in the numerator.

• SOLUTION MAP
Follow the Solution Map to Solve the Problem

SOLUTION

\[
194 \text{ cm} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in.}} = 6.3648 \text{ ft}
\]
When converting quantities with units raised to a power, the conversion factor must also be raised to that power.
Conversion with Units Raised to a Power

We cube both sides to obtain the proper conversion factor.

\[(2.54 \text{ cm})^3 = (1 \text{ in.})^3\]

\[(2.54)^3 \text{ cm}^3 = 1^3 \text{ in.}^3\]

\[16.387 \text{ cm}^3 = 1 \text{ in.}^3\]

We can do the same thing in fractional form.

\[1255 \text{ cm}^3 \times \frac{1 \text{ in.}^3}{16.387 \text{ cm}^3} = 76.5851 \text{ in.}^3 = 76.59 \text{ in.}^3\]
Physical Property: Density

- Why do some people pay more than $3000 for a bicycle made of titanium?
- For a given volume of metal, titanium has less mass than steel.
- We describe this property by saying that titanium (4.50 g/cm$^3$) is less dense than iron (7.86 g/cm$^3$).
Density

The density of a substance is the ratio of its mass to its volume.

\[
\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{or} \quad d = \frac{m}{V}
\]
We calculate the density of a substance by dividing the mass of a given amount of the substance by its volume.

For example, a sample of liquid has a volume of 22.5 mL and a mass of 27.2 g.

To find its density, we use the equation \( d = \frac{m}{V} \).

\[
\begin{align*}
d &= \frac{m}{V} = \frac{27.2 \text{ g}}{22.5 \text{ mL}} = 1.21 \text{ g/mL}
\end{align*}
\]
A Solution Map Involving the Equation for Density

- In a problem involving an equation, the solution map shows how the *equation* takes you from the *given* quantities to the *find* quantity.

\[ d = \frac{m}{V} \]
Density as a Conversion Factor

- We can use the density of a substance as a conversion factor between the mass of the substance and its volume.

- For a liquid substance with a density of 1.32 g/cm$^3$, what volume should be measured to deliver a mass of 68.4 g?
Density as a Conversion Factor

Solution Map

\[
\begin{align*}
\text{g} & \quad \text{cm}^3 \\
\frac{1 \text{ cm}^3}{1.32 \text{ g}} & \quad \frac{1 \text{ mL}}{1 \text{ cm}^3}
\end{align*}
\]

Solution

\[
68.4 \text{ g} \times \frac{1 \text{ cm}^3}{1.32 \text{ g}} \times \frac{1 \text{ mL}}{1 \text{ cm}^3} = 51.8 \text{ mL}
\]

Measure 51.8 mL to obtain 68.4 g of the liquid.
Table 2.4 provides a list of the densities of some common substances. These data are needed when solving homework problems.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Density (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>charcoal, oak</td>
<td>0.57</td>
</tr>
<tr>
<td>ethanol</td>
<td>0.789</td>
</tr>
<tr>
<td>ice</td>
<td>0.92</td>
</tr>
<tr>
<td>water</td>
<td>1.0</td>
</tr>
<tr>
<td>glass</td>
<td>2.6</td>
</tr>
<tr>
<td>aluminum</td>
<td>2.7</td>
</tr>
<tr>
<td>titanium</td>
<td>4.50</td>
</tr>
<tr>
<td>iron</td>
<td>7.86</td>
</tr>
<tr>
<td>copper</td>
<td>8.96</td>
</tr>
<tr>
<td>lead</td>
<td>11.4</td>
</tr>
<tr>
<td>gold</td>
<td>19.3</td>
</tr>
<tr>
<td>platinum</td>
<td>21.4</td>
</tr>
</tbody>
</table>
• A titanium bicycle frame contains the same amount of titanium as a titanium cube measuring 6.8 cm on a side.
• Use the density of titanium to calculate the mass in kilograms of titanium in the frame.
• What would be the mass of a similar frame composed of iron?

Example: Comparing Densities
Uncertainty:

- Scientists report measured quantities so that the number of digits reflects the certainty in the measurement.
- Write measured quantities so that every digit is certain except the last, which is estimated.
Units:

• Measured quantities usually have units associated with them.
• The SI units:
  length: meter, mass: kilogram, time: second
• Prefix multipliers such as *kilo*- or *milli*- are often used in combination with these basic units.
• The SI units of volume are units of length raised to the third power; liters or milliliters are often used as well.
Density:

• The density of a substance is its mass divided by its volume, $d = \frac{m}{V}$, and is usually reported in units of grams per cubic centimeter or grams per milliliter.

• Density is a fundamental property of all substances and generally differs from one substance to another.
Chemical Skills Learning Objectives

1. LO: Express very large and very small numbers using scientific notation.
2. LO: Report measured quantities to the right number of digits.
3. LO: Determine which digits in a number are significant.
4. LO: Round numbers to the correct number of significant figures.
5. LO: Determine the correct number of significant figures in the results of multiplication and division calculations.

6. LO: Determine the correct number of significant figures in the results of addition and subtraction calculations.

7. LO: Determine the correct number of significant figures in the results of calculations involving both addition/subtraction and multiplication/division.
8. LO: Convert between units.

9. LO: Convert units raised to a power.

10. LO: Calculate the density of a substance.

11. LO: Use density as a conversion factor.
In 1999, NASA lost a $94 million orbiter because two groups of engineers failed to communicate to each other the units that they used in their calculations. Consequently, the orbiter descended too far into the Martian atmosphere and burned up.
Highlight Problem Involving Units

• Suppose that the Mars orbiter was to have established orbit at 155 km and that one group of engineers specified this distance as $1.55 \times 10^5$ m.

• Suppose further that a second group of engineers programmed the orbiter to go to $1.55 \times 10^5$ ft.

• What was the difference in kilometers between the two altitudes?

• How low did the probe go?